IMPROVED PI-NETWORK MEASUREMENT OF ALL QUARTZ CRYSTAL PARAMETERS BY APPROPRIATE MODELLING OF THE TEST FIXTURE

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1. Abstract

The measurement of quartz crystal units is standardized in IEC 444. This means, that different test fixtures and additional elements must be used, e.g. for the determination of the load resonance parameters (IEC 444-4) or for physical C_0-compensation (IEC 444-3). The method to be presented in this paper introduces a general approach, which allows the precise determination of all quartz crystal parameters with one test fixture only by appropriate modelling :
- inductances in series to the inserted crystal modelling the electrical connection
- a capacitance between the PI-network ports modelling the crosstalk
- complex impedance of the PI-network.

All these elements are computed by a calibration routine with an open, a short and a calibration resistor in the complete frequency range. The quartz crystal parameters are determined by complex mathematical algorithms for the measurement data. Measurement results with different hardware will be presented in comparison to the standardized hardware based methods of IEC 444.

2. Introduction

The measurement of quartz crystal parameters is the objective of TC 49, Working Group 6 of IEC. The real problem is to find good correlation between standardized measurement methods and the requirements of the application in quartz crystal oscillators and filters, where the quartz crystal is object of circuit impedances (variable for frequency modulation) and drive levels.

Furthermore the equivalent circuit of the quartz crystal unit must be modified for some applications. In this paper the "standard circuit" is used with a series circuit of the dynamic capacitance C_1 and inductance L_1 and the series resonance resistance R1, shunted with a capacitance (static capacitance C_0 - measured at a low frequency [10 MHz ... 20 MHz] and parallel capacitance C_p - effective at the series resonance frequency f_s), given in figure 2.

The actual standard methods for the measurement of quartz crystals are given by IEC 444-1 to 444-4 with the use of a well defined PI-network, assumed to be purely resistive, and to cover the frequency range up to 200 MHz.

Due to new requirements of quartz crystal units and surface acoustic resonators (SAWR) with frequencies up to 1 GHz, new enclosure types (e.g. for SMT), and the calibration problems (especially for the reference resistors and the load capacitors), in the last years different new measurement methods were published and are in discussion for standardization (e.g. the s-parameter method or software based error corrected methods with the PI-network).

The aim of the approach given in this paper is to overcome the problems with the actual standardized methods by software solutions and to use further the existing hardware with one PI-network only.

3. Measurement Method

Definitions : f : frequency (Hz)
ω : circular frequency, $\omega = 2\pi f$

This measurement method complies with the passive transmission method according to IEC 444 with a PI-network.
The essential differences are :
- modelling of the PI-network with crosstalk capacitance C_c, series inductance L_s and complex termination T
- extended calibration routine for the determination of C_c, L_s and complex T
- definition of the parallel capacitance C_p at the series resonance frequency
- mathematical compensation of C_p in the complete frequency range instead of physical compensation with a tuned parallel circuit for frequencies above 80 MHz
- determination of the load resonance frequency f_L by an amplitude search instead of the use of physical load capacitors

3.1 Measurement circuit

The measurement circuit is given in figure 1 and complies with IEC 444

Proceedings of the 6th European Frequency and Time Forum, held at ESTEC, Noordwijk, NL, 17–19 March 1992 (ESA SP-340, June 1992).

3.2 Model of the PI-network

The model is shown in figure 1, too, with :

T : complex termination resistance of the PI-network (IEC 444 : T = 25 Ω real)

C_c : crosstalk capacitance between the PI-network-ports

L_s : series inductance of the connection to PI-network ports

Typical value for C_c is tween 0.1 pF and for L_s between 0,2 nH and up to 20 nH (for a distance of 5 cm between the PI-network and the measurement plane). They should be determined at frequencies above 10 MHz. The influence of C_c and L_s cannot be neglected for the determination of C_o or C_p, for the load resonance frequency f_L and for all parameters of high frequency quartz crystals.
The complex resistance X, shown in figure 1, is calculated with:

$$X = \left(\frac{K}{V} - 1\right) * T$$

The resistance Z of the device under test (see figure 1) is computed with :

$$Z = \frac{1}{\frac{1}{X} - j\omega C_c} - j\omega L_s$$

3.3 Calibration of the PI-network

Definitions : $L = 2\pi f L_s$

$$C = \frac{1}{2\pi f C_c}$$

For precise measurements care should be taken for good measurement setup (low contact resistances, well defined double shielded cables, averaging of measurement data).
Three measurements are taken with logaritmic frequency scaling in the interesting frequency range (e.g. 0.5 MHz to 200 MHz) :

- open ($Z = \infty$)
- short ($Z = 0$)
- with known calibration resistor (Z = R, complex, real part approx. 25 Ω)

giving the following results :

Z	V	X	calculation
∞	P	$X_P = -jC$	$= (\frac{K}{P} - 1) * T$
0	S	$X_S = +j\frac{C*L}{C - L}$	$= (\frac{K}{S} - 1) * T$
R	E	$X_E = \frac{L*C - jR*C}{R + j(L - C)}$	$= (\frac{K}{E} - 1) * T$

Column V gives the voltage ratio of the outputs of the measurement channel U_M and the reference channel U_R, defined as P,S, and E for open, short, and calibration resistor.

These three complex equations for the two complex variables K and T and the two real variables C and L can be solved principally, but not explicitely, for each frequency.

Eliminating T from these three equations results in two equations :

$$X_P - X_S = \left(\frac{1}{P} - \frac{1}{S}\right) * K * T$$

$$X_E - X_S = \left(\frac{1}{E} - \frac{1}{S}\right) * K * T$$

This can be reduced to one equation by eliminating K * T :

$$\frac{X_P - X_S}{\left(\frac{1}{P} - \frac{1}{S}\right)} - \frac{X_E - X_S}{\left(\frac{1}{E} - \frac{1}{S}\right)} = 0$$

Inserting the above resistances X_P, X_E and X_S gives for each measurement frequency the final complex equation with the unknown values C_c, L_s, the known (complex) reference resistor R and the measured voltage ratios P, S and E.

As this equation cannot be solved explicitely, the following algorithm is applied :

- calculate a first approximation for C_c with K = S , T = 25 Ohm and

$$C_c = \frac{1}{25 * \omega * IMAG\left(\frac{K}{P} - 1\right)}$$

- set a first approximation for L_s = 1 nH
- vary C_c and L_s in a double loop with diminuishing steps for C_c and L_s
- sum the squares of the complex equation error for each frequency above 10 MHz
- look for a minimum sum of the squares, resulting in the final values of C_c and L_s.

With the equations for the measurements with short and reference resistor the complex termination resistance T is found for each frequency :

$$T = \frac{X_S * S - X_E * E}{E - S}$$

T is then mathematically smoothed in both real and imaginary part by substituting their values with the second order parabola values calculated with the next five frequencies.

With the equation for the measurements with short the complex reference voltage ratio K is found for each frequency :

$$K = \left(\frac{X_S}{T} + 1\right) * S$$

K is then mathematically smoothed as given above for T.

3.4 Calibration of reference resistors

Excellent reference resistors with traced calibration data are commercially available e.g. from Wandel & Goltermann (W&G), Germany. Their model consists of an resistance in series with an inductance, shunted by a capacitance. Typical values are 25 Ω, 0.1 pF and 0.4 nH.

With a similar procedure as in clause 3.3 reference resistors can be calibrated together with the PI-network.

From the general equation of 3.2 the termination resistance T can be expressed by :

$$T = \frac{X}{\frac{K}{V} - 1}$$

Applying this formula to a pair of resistors X_1 and X_2 with the voltage ratios V_1 and V_2 and then eliminating K, the following equation is obtained for T :

$$T = \frac{X_1 * V_1 - X_2 * V_2}{V_2 - V_1}$$

Applying above formula to an other pair of resistances X_1 and X_3 with the voltage ratios V_1 and V_3 and eliminating then K an other equation is obtained for T :

$$T = \frac{X_1 * V_1 - X_3 * V_3}{V_3 - V_1}$$

Combining these 2 equations yields then :

$$\frac{X_1 * V_1 - X_2 * V_2}{V_2 - V_1} - \frac{X_1 * V_1 - X_3 * V_3}{V_3 - V_1} = 0$$

This equation can then be minimized with respect to the equivalent values of the calibration resistors E_1, E_2 and E_3 and the PI-network values of C_c and L_s. The calibration resistors are chosen as a short (E_1) and two resistors of about 25 Ω (E_2) and 75 Ω (E_3). Apart from the measurements with the calibration resistors an "open" measurement is performed.

The equation is solved with the following algorithm :
- define the equivalent circuit of the calibration resistors E_2 and E_3
- measure with a standard meter the static ohmic values of E_2 and E_3
- set the inductances of E_2 and E_3 to zero
- set the capacitances of E_2 and E_3 very small (< 1 fF).
- calculate a first approximation for C_c with K = S, T = 25 Ω and

$$C_c = \frac{1}{25 * \omega * IMAG\left(\frac{K}{P} - 1\right)}$$

- set a first approximation for L_s = 1 nH
- vary C_c, L_s and the parameters of E_2 and E_3 in a multiple loop with diminuishing steps for these parameters
- sum the squares of the complex equation error for each frequency above 10 MHz
- look for a minimum sum of the squares, resulting in the final values of C_c, L_s and the parameters of E_2 and E_3
- calculate then T and K as in 3.3.

4. Equivalent circuit of a quartz crystal unit

The equivalent circuit is given in figure 2 with some definitions. The only difference to IEC 444 is the definition of the parallel capacitance C_p besides C_0, due to the fact, that the internal structure of the quartz crystal unit results in impedance transformations which cannot be neglected for precise measurements above about 30 MHz. Therefore C_p - effective at f_s - is no longer identical to C_0 measured at a lower frequency.

The complex resistance Z of a quartz crystal is given by :

$$Z = \frac{1}{\frac{1}{R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)} + j\omega C_p}$$

5. Measurement of the equivalent parameters of a quartz crystal

The measurement is performed in five steps :
- static capacitance C_0
- parallel capacitance C_p
- coarse search of the series resonance frequency f_s
- at least two measurements close to f_s and determination of the series circuit parameters C_1, R_1, L_1, f_s, Q, Q_{eff}
- search of the load resonance frequency f_L (if applicable)

5.1 Measurement of the static capacitance C_0

At frequencies far from resonances the resistance Z of the crystal is given by:

$$Z = -j\frac{1}{\omega C_0}$$

The effective resistance X inserted in the PI-network is then :

$$X = \frac{1}{\frac{1}{\left(-j\frac{1}{\omega C_0} + j\omega L_s\right)} + j\omega C_c}$$

The measurement is performed at five equidistant frequencies between 10 MHz and 15 MHz, resulting in a matrix MC0. The mean and spread values of C_0 are : C_{0mean} and $C_{ospread}$

The condition for validity of the C_0-measurement is :

$C_{0spread}$ < 0.1 pF OR $C_{ospread}$ / C_{omean} < 0.03

If this condition is not satisfied, the non-valid values are eliminated by :

ABS (C_{omean} - MC0) > 3 * $C_{ospread}$

and further appropriate measurements are taken.

Remark : These eliminations are necessary, when values are taken at frequenecies close to crystal resonances (other overtones or spurious modes)

5.2 Measurement of the parallel capacitance C_p

a) f_N <= 30 MHz :

$C_p = C_0$

b) f_N > 30 MHz :

Measurement at the frequency $f_p = f_N$ - 1 MHz, with $\omega_p = 2\pi f_p$:

$$Z = -j\frac{1}{\omega_p C_p}$$

$$X = \frac{1}{\frac{1}{\left(-j\frac{1}{\omega_p C_p} + j\omega_p L_s\right)} + j\omega_p C_c}$$

Correction after measurement of f_s and C_1 :

$$C_p = C_p - \frac{C_1}{1 - \left(\frac{f_p}{f_s}\right)^2}$$

5.3 Coarse determination of f_s : f_c

a) With network analyzer :
Fast sweep over the specified frequency range and search of the frequencies with maximum amplitudes :

f_{up} for sweep direction up

f_{do} for sweep direction down

$$f_c = \frac{f_{up} + f_{do}}{2}$$

b) With vector voltmeter :

f_N : nominal frequency of the quartz crystal

two measurements

$$f_N * \left(1 + \frac{1}{Q_{eff}}\right) \text{ and } f_N * \left(1 - \frac{1}{Q_{eff}}\right)$$

where Q_{eff} is an estimated value

calculation of f_c according to clause 5.4.

5.4 Measurement of f_s, R_1, C_1, L_1, Q, Q_{eff}

Measurement of the resistances Z_S of the series circuit at two frequencies f_x und f_y with the following presettings from other crystals of the batch or from standard values :

Q_{eff} = expected or estimated typical value

$$f_x = f_c * \left(1 + \frac{1}{2*Q_{eff}}\right)$$

$$f_y = f_c * \left(1 - \frac{1}{2*Q_{eff}}\right)$$

$$TIME = \frac{4*Q_{eff}}{\pi f_c}$$ rise time for oscillation of the quartz crystal

The resistance of the series circuit of the quartz crystal is :

$$Z_S = \frac{1}{\frac{1}{Z} - j\omega C_p} = R_1 + j2\pi f L_1 - j\frac{1}{2\pi f C_1}$$

Using the definition for the series resonance frequency f_s one obtains :

$$-j\frac{1}{2\pi f_s C_1} = -j2\pi f_s L_1 \quad \text{and}$$

$$Z_S = R_1 + j2\pi L_1 * \left(f - \frac{f_s^2}{f}\right)$$

For $|f - f_s| << f_s$ it can be written:

$$f - \frac{f_s^2}{f} = (f - fs)$$

and Z_S can then be expressed by :

$$Z_S = R_1 + j4\pi L_1 * (f - f_s)$$

The measured real and imaginary parts of Z_S at f_x and f_y are :

R_X, I_X, R_Y, I_Y

$$R_X + jI_X = R_1 + j4\pi L_1 * (f_x - f_s)$$

$$R_Y + jI_Y = R_1 + j4\pi L_1 * (f_y - f_s)$$

From these equations calculations are done for :

$$R_1 = \frac{R_X + R_Y}{2}$$

$$L_1 = \frac{I_X - I_Y}{4\pi(f_x - f_y)}$$

$$f_s = \frac{f_x + f_y}{2} + (f_x - f_y) * \frac{I_X + I_Y}{I_Y - I_X}$$

$$C_1 = \frac{1}{(2\pi f_s)^2 * L_1}$$

$$Q = \frac{2\pi f_s L_1}{R_1} \qquad Q_{eff} = Q * \frac{R_1}{R_1 + ABS(T)}$$

Correction of C_p due to the vicinity of f_c to f_s for $f_N >= 30$ MHz :

second calculations for R_1, L_1, f_s, C_1, Q and Q_{eff} with

$$C_p = C_p - \frac{C_1}{1 - \left(\frac{f_p}{f_s}\right)^2}$$

Condition for this measurement :

$$4 * \frac{Q_{eff}}{\pi f_s} < TIME \text{ AND } f_x - f_y < \frac{f_s}{Q_{eff}} \text{ AND}$$

$$[ABS(f_s - f_x) < f_x - f_y \text{ OR } ABS(f_s - f_y) < f_x - f_y]$$

If this condition is not satisfied, a further measurement is done with :

$$TIME = 6 * \frac{Q_{eff}}{\pi f_s}$$

$$f_x = f_s + \frac{f_s}{2Q_{eff}}$$

$$f_y = f_s - \frac{f_s}{2Q_{eff}}$$

5.5 Measurement of load resonance : frequency f_L and resi stance R_L

Definition of the load resonance resistance R_L :

$$R_L = R_1 * \left(1 + \frac{C_p}{C_L}\right)^2$$

Definition of the load resonance frequency f_L :

At f_L the imaginary part of the impedance of the quartz crystal is oppositely equal to the reactance of the load capacitor C_L :

$$Z_L = R_L + j\frac{1}{2\pi f_L C_L}$$

The load resonance frequency can be calculated in a first approximation with :

$$f_L = f_s * \left(1 + \frac{C_1}{2*(C_L + C_p)}\right)$$

With this new approach a physical load capacitor is not used, f_L is determined by an amplitude search of Z_L.

In order to get an accuracy of 1 ppm for f_L of fundamental mode crystals, the amplitudes must be measured with an accuracy of 0.1 %, which requires a good measurement circuitry and careful calibration.

As the load resonance frequency f_L is far from the series resonance frequency f_s, the crystal impedance is very high and mainly reactive. Therefore the output power of the generator must be increased in order to give the required drive level P_N at f_L.

If the generator cannot deliver the necessary high output power for f_L, the measurements of both, f_s and f_L, must be repeated at an accordingly lower drive level, which can be realized at f_L. The difference between these two frequencies must then be added to the series resonance frequency at the nominal drive level in order to get the correct load resonance frequency. With this method, the effects of drive level dependence (DLD) of the crystal is encountered.

The output power of the generator is calculated as follows :
For series resonance :

- current through series circuit : $I_S = \sqrt{\frac{P_N}{R_1}}$
- voltage across crystal : $U_Q = I_S * R_1$
- current through C_p : $I_C = U_Q * j2\pi f_s C_p$
- current through crystal : $I_Q = I_S + I_P$
- voltage across C_c : $U_P = U_Q + I_Q * j2\pi f_s L_s$
- current through C_c : $I_P = U_P * j2\pi f_s C_c$
- current through PI-network : $I_T = I_Q + I_P$
- voltage across PI-network : $U_S = U_P + I_T * T$

For load resonance :

- current through crystal : $I_Q = \sqrt{\frac{P_N}{R_L}}$
- voltage across crystal : $U_Q = I_S * \left(R_L + j\frac{1}{2\pi f_L C_L}\right)$
- voltage across C_c : $U_P = U_Q + I_Q * j2\pi f_L L_s$
- current through C_c : $I_P = U_P * j2\pi f_s C_c$
- current through PI-network: $I_T = I_Q + I_P$
- voltage across PI-network : $U_L = U_P + I_T * T$

$$\text{Delta\_dBm} = 20 * \lg\left(\text{ABS}\left(\frac{U_L}{U_S}\right)\right)$$

6. Experimental results

To verify the theoretical evaluations, and to proof the expected accuracy improvement, 70 quartz crystal units at 14 different frequencies in the range 1 MHz to 150 MHz were measured at two different places (at the facilities of KVG and at those of TELE QUARZ [TQG]) with different measurement equipment and test fixtures, and with different software packages, which used different algorithms.

The KVG equipment used a network analyzer HP 3577, a coaxial PI-network with the traditional construction as depicted in IEC 444-1.
The PI-network set-up was modelled as described in clause 3 of this paper, i.e. with modelled series inductance L_s and crosstalk capacitance C_c as in Fig.1.

The TQG equipment consisted of a generator (PTS 160), a vector voltmeter (Rohde & Schwarz ZPV) and a PI-network in hybrid construction. The PI-network was modelled by a complex termination impedance R_T instead of modelling discrete circuit elements (as above). The crosstalk capacitance C_c is assumed to be directly across Z.

The results were compared and are depicted in the following graphs:

Fig. 3 shows the comparison of the measured resonance resistance values R_r (resp.the series resonance resistance R_1).

Fig.4 shows the deviation of the measured dynamic capacitance C_1.

Fig.5 shows the deviation of the static capacitance C_0.

Fig. 6 shows the differences in the measured (series) resonance frequency

The results of all these data shows an agreement in the order of +-1ppm for frequencies and a few percent for the other parameters.

The measurement of the load resonance frequency was performed in two different ways:
- measurement with physical load capacitors according to IEC report 444-4, where KVG and TQG were using their own capacitive elements, both calibrated according to their in-house standard procedures; and
- measurement with the "mathematical approach" given in clause 5.5

In Fig. 7 the results for a load capacitance of 30 pF ist shown in three different curves:
- comparison "phys-phys", i.e. using physical load capacitors at both places,
- comparison "math-math", i.e. using the mathematical methods at both places;, and
- comparison "phys-math", i.e. difference between "physical" and mathematical" measurement at one location (KVG).

Fig. 8 the same results are shown for a load capacitance of 10pF, which is extremely sensitive, especially for high fundamental mode frequency crystals.

7. Conclusion

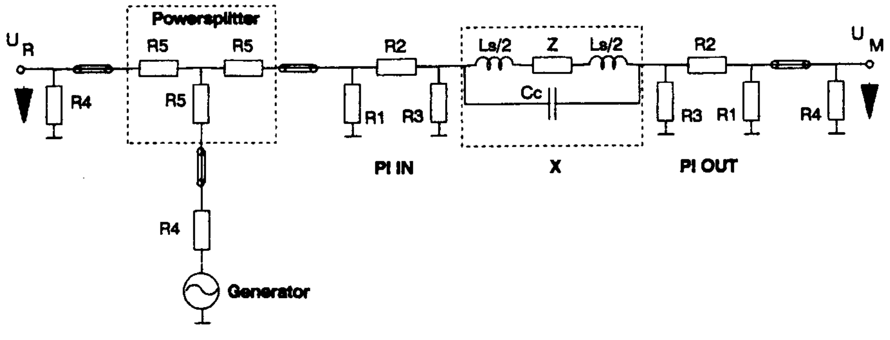
The graphs show quite clearly, that rather large frequency differences occur with physical load capacitors, which is an every-day experience of every crystal manufacturer and user. Contradictory to that, the frequency differences are reduces by a factor of 5 to 10, if the mathematical method is used.

The residual error resp. uncertainity with the mathematical methods comes down to a few ppm, only slightly more than the differences seen at the (series) resonance measurements.

These excellent results are achieved inspite of the fact, that both measurement methods were not identical in every detail, both, hardware and software.

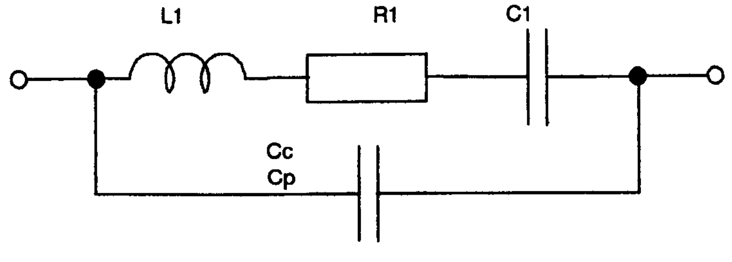
These results encourage us to propose the new method for implementation into the IEC standards. Regarding the good results of load resonance measurement, the new method will - and has to - replace the old IEC 444-4 report within short time.

Acknowledgements:
The authors wish to acklowledge R. Zimmermann (TQG) and H. Dimmler (KVG), who have done the experimental work and the evaluation of the measurement data.



R1: 159 Ohm, R2: 66.2 Ohm, R3: 14.2 Ohm, R4: 50 Ohm, R5: 50/3 Ohm
UR: Complex voltage in the reference channel
UM: Complex voltage in the measuremenı channel
V: Complex voltage ratio; V = UM / UR with resistance X
Reference voltage ratio; K = UM / UR with resistance X = 0 Ohm, short over PI - network ports

Figure 1: Measurement circuit



fN: nominal frequency of the quartz crystal unit
fs: series resonance frequency, fs = 1 / 2 / Pi / (L1 * C1) ^ 0.5
Q: Q - value, Q = 2 * Pi * fs * L1 / R1 = 1 / 2 / Pi / fs / C1 / R1
Qeff: effective Q - value, Qeff = Q * R1 /(R1+ABS(T))

Figure 2: Equivalent twoport circuit of a quartz crystal unit

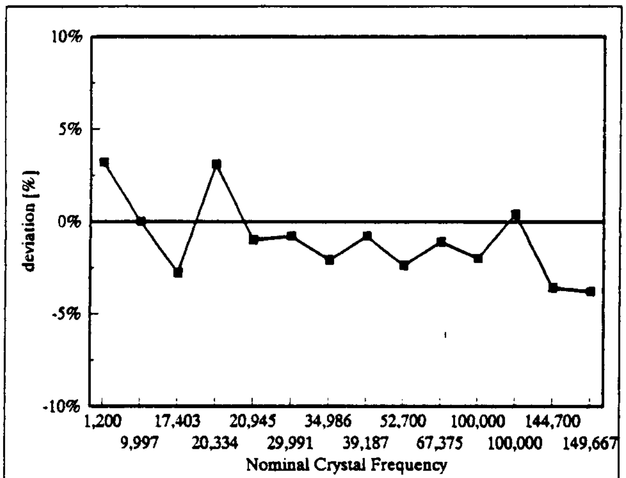


Figure 3: Comparision of Measurements Rr,R1 from KVG and TQG

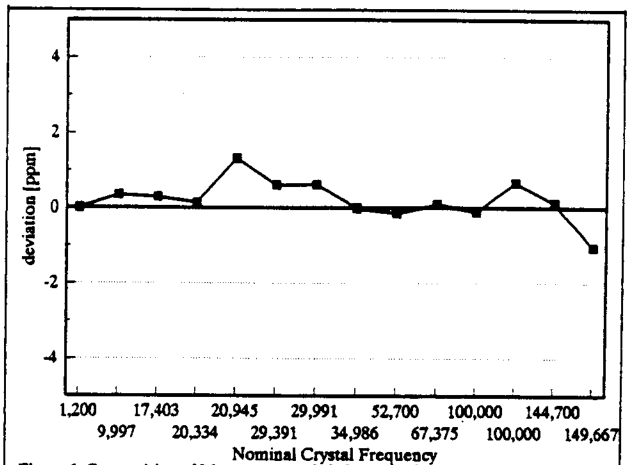


Figure 6: Comparision of Measurements fr,fs from KVG and TQG

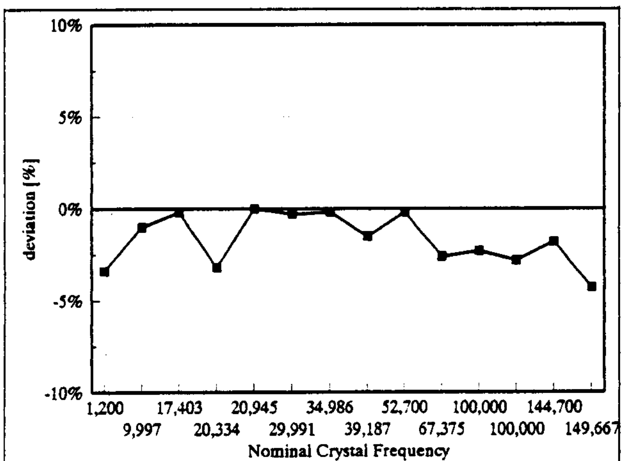


Figure 4: Comparision of Measurements C1 from KVG and TQG

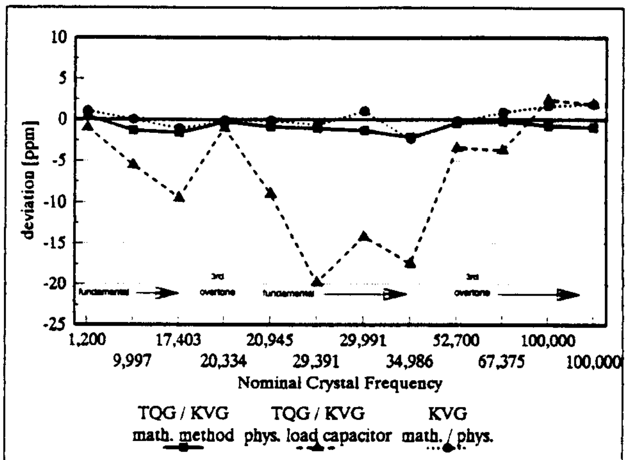


Figure 7: Comparision of Measurements Load Resonance Freq. 30pF from KVG and TQG

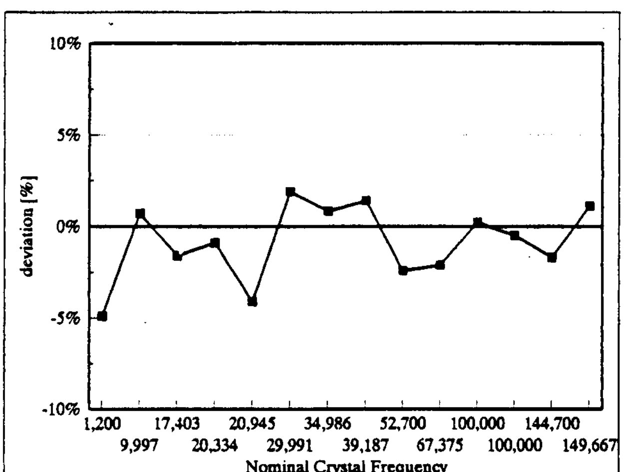


Figure 5: Comparision of Measurements C0 from KVG and TQG

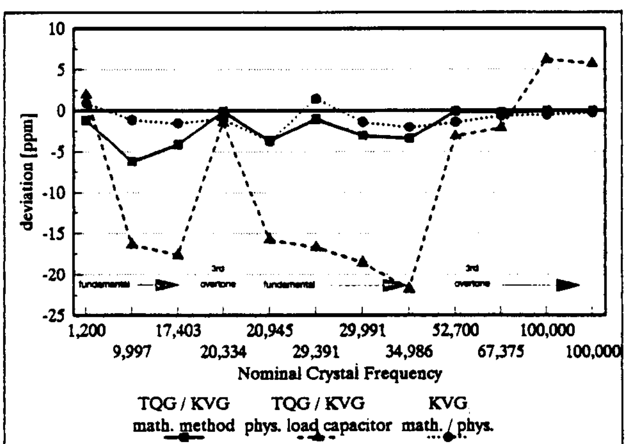


Figure 8: Comparision of Measurements Load Resonance Freq. 10pF from KVG and TQG